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Abstract

This article challenges the conventional result that a tighter environmental tax has no long-run effect on human capital accumulation in the presence of pollution arising from final output production. It demonstrates that the technology used in the abatement sector determines the existence and the direction of the growth-effect. A tighter environmental tax rises *(respectively reduces)* human capital accumulation in the presence of pollution arising from final production, if the abatement sector is relatively more intensive in human *(resp. physical)* capital than final sector. That result always holds for finite lifetime but for infinite lifetime it only holds when labor supply is endogenous.

The transitional impact of a tighter environmental policy is also investigated.

Keywords : Growth; Environment; Overlapping generations; Human capital; Abatement.

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1 Introduction

The role of the human capital accumulation on economic growth has been substantially investigated.[1] Nevertheless, the influence of the environmental taxation on education and growth has not been studied extensively at theoretical level. Moreover, the few existing articles find that environmental tax has no long-run impact on human capital accumulation when final output is the source of pollution (Gradus and Smulders, 1993; Hettich, 1998). Our article re-examines this finding *(i)* by extending the analysis of the environment-growth nexus with education to finitely-lived agents and *(ii)* by relaxing the basic assumption of a similar production technology in final output and abatement sectors. It demonstrates that the technology used in the abatement sector determines the existence and the direction of the growth-effect. Therefore it improves our understanding of how environmental tax impacts long-run growth when education is the channel of transmission.

This article is in line with the seminal contribution of Gradus and Smulders (1993). In a model à la Lucas (1988) where physical capital is the source of pollution, they demonstrate that the environment promotes the steady-state growth rate when pollution depreciates the stock of human capital. Van Ewijk and Van Wijnbergen (1995) obtain a similar result by considering that pollution reduces the ability to train. Therefore, they conclude that the environmental tax never influences human capital accumulation in the long-run if educational activities are not directly affected by pollution. Hettich (1998) introduces endogenous labor supply assuming no direct impact of pollution on human capital accumulation. He finds that the environmental tax promotes education and growth, through the channel of labor supply. Because the increase in the environmental tax compels firms to increase their abatement activities at the expense of the household's consumption, households substitute leisure to education to counteract this negative effect, and the growth rate rises. However, Hettich (1998) demonstrates that his result holds only when pollution arises from physical capital. When final output is the source of pollution, a tighter environmental policy reduces both the returns to physical capital and the wage rate that is a part of the returns to education. The incentives of agents to invest more in education vanish and the long-run growth rate is not affected by pollution tax.

Recently, Pautrel (2011) re-examines those findings by enlarging the infinite lifetime model of Gradus and Smulders (1993) to finitely-lived agents. He demonstrates that the

[1] For more details, see for example, the texbook of Acemoglu (2009), among others.

result found by the two authors does not hold anymore. When physical capital is the source of pollution and agents have finite lifetime, a tighter environmental tax enhances human capital accumulation in the long-run whereas pollution does not affect educational activities and labor supply is inelastic. Finite lifetime introduces a turnover of generations that disconnects the aggregate consumption growth to the interest rate and promotes the investment in human capital accumulation. Nevertheless, the author does not extend his analysis to the case of a pollution arising from final production.

An another important recent contribution is the one by Grimaud and Tournemaine (2007). They study the role of education in the environment-growth nexus, in a model with R&D and human capital accumulation à la Lucas (1988). They demonstrate that a tighter environmental policy promotes growth when education directly enters the utility function as a consumption good. They depart from the basic structure of a similar technology between output sector and abatement sector by modeling a R&D sector aimed at creating knowledge to reduce the flow of pollution emissions. A higher environmental tax rises the price of the good whose production pollutes and therefore the relative cost of education diminishes. Agents reduce their investment in human capital accumulation and because education is the engine of growth, the growth rises at the steady-state. As highlighted by the authors, the way education influences utility is crucial to their results and the more realistic way they model abatement technology is important as well. It is the reason why the role played by the technology of abatement sector in the environment-growth nexus with education must be studied.

In the present article, we re-examine the environment growth-nexus with education when pollution arises from final output assuming that lifetime is finite and relaxing the basic assumption of a similar technology in final output and abatement sectors.[2] We use the Yaari (1965)-Blanchard (1985) overlapping generations model with environment and we study both long-run and short-run outcome of a tighter environmental tax.

The contribution of this article is threefold. First, with the basic assumption of similar technology across sectors, we demonstrate that the results found by Gradus and Smulders (1993) and Hettich (1998) can be generalized to finite lifetime: the environmental tax has

[2]For the sake of simplicity, we will just take into account the differences of relative factor intensity in final output and abatement sectors. More complex and more realistic modeling of abatement activities (like Grimaud and Tournemaine, 2007) is out of the scope of this article. Our results should be easily extended to such a modeling.

no growth effect in the long-run when final output is the source of pollution. That finding is conflicting with the result obtained by Pautrel (2011) that the generational turnover effect introduced by finite-lifetime leads the environmental policy to promote human capital in the long-run when pollution arises from physical capital. Our result may be explained by the fact that both the interest rate and the wage rate are reduced by a tighter environmental tax on output. Therefore, the positive impact of the generational turnover effect on human capital accumulation found by Pautrel (2011) is offset by the reduction of the returns to education so that the positive impact of the environmental tax on growth vanishes.

Second, we demonstrate that, relaxing the basic assumption of similar factor intensities in final output and abatement sectors, leads to an impact of the environmental tax on the long-run growth. The environmental tax will boost (*respectively harm*) long-run human capital accumulation when the abatement sector is relatively more intensive in human capital (*resp. physical capital*) than the output sector. That result comes from the fact that tighter environmental tax increases abatement activities and generates factor reallocations between sectors. Because factorial intensities differ across sectors, that leads to a relative scarcity of the factor that is intensively used in the abatement sector. When the abatement sector is relatively more intensive in human capital (*resp. physical capital*), the reward of human capital relatively to the reward of physical capital rises (*resp. diminishes*). Agents are incited to invest more (*resp. less*) in education and human capital accumulation increases (*resp. decreases*). We also demonstrate that our result always holds for finite lifetime but it holds for infinite lifetime only when labor supply is endogenous. Therefore, the findings of Gradus and Smulders (1993) in the case of exogenous labor supply and a similar technology in abatement and output sectors remain valid when technologies between sectors differ.[3] Because, it is empirically relevant to consider that the abatement sector is more intensive in human capital than the output sector, our results suggest that, even if the source of pollution is final output, tighter environmental tax will enhance the long-run human capital accumulation as long as lifetime is finite and/or labor supply is endogenous.

Finally, we investigate whether the role played by the difference of technology in final output and abatement sectors also exists when pollutant emissions originate from physical capital. We demonstrate that, in such a case, the environmental policy always promote

[3]The reason comes from the exogenous nature of the efficiency of time allocated to educational activities. With exogenous labor supply, the expression of the time invested in education is similar to the one find by Lucas (1988) and it is therefore independent from the environmental tax.

long-run human capital accumulation whatever the technologies used in final output sector and in the abatement sector. As previously mentioned, when the abatement sector is relatively more intensive in physical capital, the reward of physical capital relatively to the reward of human capital increases. Nevertheless, compared with the case where output is taxed, this increase is lower because the environmental policy only diminishes the interest rate and not the wage rate. Therefore the downward pressure of the environmental tax on the interest rate always compensate the aforementioned increase in the relative reward of physical capital, so that the global impact is a higher human capital accumulation.

The article is structured as follows. Section 2 gives the basic framework of the model. Section 3 investigates the long-run influence of the environmental tax on the economy and its impact during the transition. Section 4 studies the case where pollution arises from physical capital. Section 5 concludes.

2 The general framework

2.1 The households' behaviour

We use the Yaari (1965)-Blanchard (1985) overlapping generations model with human capital accumulation and environmental concerns. Time is continuous. Each individual born at time s faces a constant probability of death per unit of time $\beta \geq 0$. Consequently his life expectancy is $1/\beta$. When β increases, the life span decreases. At time s, a cohort of size β is born. At time $t \geq s$, the cohort born at s has a size equal to $N(s,t) = \beta e^{-\beta(t-s)}$ and the constant population is equal to $N(t) = \int_{-\infty}^{t} N(s,t)ds = 1$. There are insurance companies and there is no bequest motive.

The expected utility function of an agent born at $s \leq t$ is:

$$\int_{s}^{\infty} \left[\log c(s,t) + \xi_l \log l(s,t) - \zeta \log S(t)\right] e^{-(\varrho+\beta)(t-s)} dt \tag{1}$$

where $c(s,t)$ denotes consumption in period t of an agent born at time s, $\varrho \geq 0$ is the rate of time preference, $S(t)$ is the stock of pollution at date t and $\zeta > 0$ measures the weight in utility attached to the environment. $l(s,t)$ is leisure time at date t of an agent born at date $s \leq t$.

The representative agent can increase his stock of human capital by devoting time to schooling, according to Lucas (1988). Because each agent allocates a part $u(s,t) \in]0,1[$ of her time to production, a part $l(s,t) \in]0,1[$ in leisure, her remaining time for education is $1-u(s,t)-l(s,t)$. The temporal evolution of the individual stock of human capital is

$$\dot{h}(s,t) = B\left[1-u(s,t)-l(s,t)\right]h(s,t) \tag{2}$$

where B is the efficiency of schooling activities and $h(s,t)$ is the stock of human capital at time t of an individual born at time s. Conveniently, we assume that the human capital of the agent when he borns, $h(s,s)$, is inherited from the dying generation. To capture the intergenerational transmission of knowledge, we follow Bovenberg and Van Ewijk (1997) considering that newborn inherit from the dying generation the average aggregate human capital stock, that is $h(s,s) = H(s)$ (population being equal to unity).[4]

Households face the following budget constraint:

$$\dot{a}(s,t) = \left[r_n(t)+\beta\right]a(s,t) + u(s,t)h(s,t)\mathrm{w}(t) - c(s,t) \tag{3}$$

where $a(s,t)$ is the financial wealth in period t, $\mathrm{w}(t)$ represents the wage rate per effective unit of human capital $u(s,t)h(s,t)$ and r_n is the after-tax interest rate.[5] In addition to the budget constraint, there exists a transversality condition which must be satisfied to prevent households from accumulating debt indefinitely:

$$\lim_{v\to\infty}\left[a_{s,v}e^{-(r_n+\beta)(v-t)}\right] = 0$$

The representative agent chooses the time path for $c(s,t)$ and his working time $u(s,t)$ by maximizing (1) subject to (2) and (3). It yields

$$\dot{c}(s,t) = \left[r_n - \varrho\right]c(s,t) \tag{4}$$

and the individual leisure choice

$$l(s,t) = \xi_l \frac{c(s,t)}{\mathrm{w}(t)\ h(s,t)} \tag{5}$$

Integrating (3) and (4) and combining the results gives the consumption at time t of an agent born at time s:

$$c(s,t) = (\varrho+\beta)\left[a(s,t)+\omega(s,t)\right]$$

[4]Assuming that $h(t,t) = \eta H(t)$ with $\eta \in]0,1[$ would not modify our qualitative results. Proof upon request.

[5]We introduce the after-tax interest rate r_n because in section 4 we will study the impact of an environmental tax on physical capital income. For the moment there is no tax on physical capital income and therefore $r_n = r$ the real interest rate.

where $\omega(s,t) \equiv \int_t^\infty [u(s,\nu)h(s,\nu)\mathrm{w}(\nu)]\, e^{-\int_t^\nu [r_n(\zeta)+\beta]d\zeta} d\nu$ is the present value of lifetime earning. It also gives:

$$\frac{\dot{w}(t)}{w(t)} + B[1 - l(s,t)] = r_n(t) + \beta \tag{6}$$

the equality between the rate of returns to human capital (the left-hand side) and the effective rate of interest (the interest rate on the debt plus the insurance premium the agent has to pay when borrowing β). It is straightforward from that relationship that the time allocated to leisure is the same for all individuals whatever her date of birth: $l(s,t) = l(t)$. All individual variables being additive across individuals, the aggregate consumption equals

$$C(t) = \int_{-\infty}^t c(s,t)\beta e^{-\beta(t-s)} ds = (\varrho + \beta)\,[K(t) + \Omega(t)] \tag{7}$$

where $\Omega(t) \equiv \int_{-\infty}^t \omega(s,t)\beta e^{-\beta(t-s)} ds$ is aggregate human wealth in the economy. The aggregate stock of physical capital is defined by

$$K(t) = \int_{-\infty}^t a(s,t)\beta e^{-\beta(t-s)} ds$$

and the aggregate human capital is

$$H(t) = \int_{-\infty}^t h(s,t)\beta e^{-\beta(t-s)} ds, \tag{8}$$

2.2 Production sectors, the Government and the Environment

There are two production sectors that operate under perfect competition: one produces final output denoted G, the other produces abatement services denoted D. National income measured in terms of final output is[6]

$$Y = G + P_D D \tag{9}$$

where P_D is the relative price of abatement services in terms of final output production.

[6]We introduce National income because we want our modeling to replicate the Hettich (1998)' model when the final output sector and the abatement sector share the same production technology. In Hettich (1998) abatement services are made with final output and therefore are taxed as a source of pollution (see below). Assuming that final output is only taxed would give the same qualitative results. Proof upon request.

We assume that production sectors are the source of pollution and the government imposes a tax $t_Y \in]0,1[$ upon National income. Furthermore, we consider that the tax revenue $t_Y Y$ is completely used by the government to fund the purchases of the abatement services. Therefore

$$t_Y Y = P_D D \tag{10}$$

From equation (9), it comes:

$$P_D D = \frac{t_Y}{1-t_Y} G \qquad \text{and} \qquad G = (1-t_Y)Y \tag{11}$$

The final output G is produced with the following technology:

$$G = (\phi K)^{\alpha} (\psi H_p)^{1-\alpha}, \qquad \text{with } \ \phi,\ \psi,\ \alpha \ \in]0,1[$$

where ψH_p is the amount of the aggregate stock of human capital devoted to production ($H_p \equiv \left[\int_{-\infty}^{t} u(s,t)h(s,t)\beta e^{-\beta(t-s)}ds\right]$) that is used in output production (which represents a part $\psi \in]0,1[$ of H_p). And ϕK is the part of the physical capital stock used in output production. Firms in the final output sector maximize profit $(1-t_Y)G - r\phi K - w\psi H_p$ by equating factor rewards to marginal productivity:

$$\begin{aligned} r &= \alpha(1-t_Y)\frac{G}{\phi K} \\ w &= (1-\alpha)(1-t_Y)\frac{G}{\psi H_p} \end{aligned} \tag{12}$$

The abatement sector produces abatement services aimed at curbing the emissions of pollution. Physical and human capital are used in the abatement sector with the following constant-returns technology:

$$D = \left[(1-\phi)K\right]^{\varepsilon} \left[(1-\psi)H_p\right]^{1-\varepsilon}, \qquad \text{with } \ \varepsilon \in [0,1]$$

Note that when $\varepsilon = \alpha$ the abatement services sector uses the same technology than output sector, that is abatement services are produced with output. When $\varepsilon = 1$ we are in the case of Michel and Rotillon (1995) where only physical capital is used in the production of abatement services. Profit maximisation in the abatement services sector gives:

$$\begin{aligned} w &= (1-\varepsilon)(1-t_Y)\frac{P_D D}{(1-\psi)H_p} \\ r &= \varepsilon(1-t_Y)\frac{P_D D}{(1-\phi)K} \end{aligned} \tag{13}$$

From equations (13) and (11), we obtain:

$$r = \frac{\varepsilon\ t_Y G}{(1-\phi)K} \qquad \text{and} \qquad w = \frac{(1-\varepsilon)(1-t_Y)\ t_Y G}{(1-\psi)H_p}$$

From (12) we have $\frac{\alpha(1-t_Y)G}{\phi K} = \frac{\varepsilon\ t_Y G}{(1-\phi)K}$, that is

$$\phi = \frac{\alpha(1-t_Y)}{\alpha + (\varepsilon-\alpha)t_Y}$$

and $\frac{(1-\alpha)(1-t_Y)G}{\psi H_p} = \frac{(1-\varepsilon)\ t_Y G}{(1-\psi)H_p}$ gives

$$\psi = \frac{(1-\alpha)(1-t_Y)}{(1-\alpha)-(\varepsilon-\alpha)t_Y}$$

When $\varepsilon = \alpha$, we obtain $\phi = \psi = 1 - t_Y$ and therefore $G = (1-t_Y)K^\alpha H_p^{1-\alpha}$, $P_D D = t_Y K^\alpha H_p^{1-\alpha}$ and $Y = K^\alpha H_p^{1-\alpha}$ like in Hettich (1998).

The stock of pollution, denoted by S, evolves according to two opposite forces. On the one hand, it increases in the net flow of pollution, the pollutant emissions to abatement services ratio Y/D. On the other hand, it decreases due to a natural rate of decay $\zeta > 0$, such that:

$$\dot{S} = f\left(\frac{Y}{D}\right) - \zeta S, \qquad \text{with } f(\cdot) > 0,\ f'(\cdot) > 0,\ f''(\cdot) > 0$$

2.3 The general equilibrium and the balanced growth path

National income is used either to finance abatment purchase (equal to $t_Y Y$), either to consume, either to invest in physical capital. Therefore, the market clearing condition is:

$$(1-t_Y)Y = C + \dot{K}.$$

with $\dot{K} = dK/dt$. Differentiating (8) with respect to time and using the fact that $u(s,t) = u$,[7] the aggregate accumulation of human capital is:

$$\dot{H} = B\left[1-u-l\right]H$$

[7]Using (12), the equalization of the rates of returns given by equation (6) implies that the rate of returns to human capital is independent of s, therefore all individuals allocate the same effort to schooling: $u(s,t) = u$.

Because in that section there is no tax on physical capital income we have $r_n = r$, and differentiating (7) with respect to time, using the expression of dK/dt, $d\Omega/dt$ and equation (4) we obtain:

$$\dot{C}/C = r - \varrho - \beta\left(\varrho + \beta\right)K/C \tag{14}$$

The last term in the right-hand side represents the *generational turnover effect* that arises because some agents die at each date and therefore the aggregate consumption growth is reduced. The generational turnover effect increases in the probability to die β: on one hand, agents die at a higher frequency (that increases the generational turnover) and on the other hand the propensity to consume out of wealth $\varrho+\beta$ rises due to the shorter horizon.

Defining $x \equiv C/K$, $z \equiv H/K$, the model is given by three following dynamical equations

$$\dot{x}/x = \left[\alpha(1-t_Y) - \Phi(t_Y)\right]\Psi(t_Y)^{\alpha-1}(z\ u)^{1-\alpha} - \varrho - \beta(\beta+\varrho)x^{-1} + x \tag{D1.1}$$

$$\dot{z}/z = B\left[1-u-l\right] - \Phi(t_Y)\Psi(t_Y)^{\alpha-1}(z\ u)^{1-\alpha} + x \tag{D2.1}$$

$$\dot{u}/u = \alpha^{-1}\left[B(1-l) - \beta - \alpha(1-t_Y)\Psi(t_Y)^{\alpha-1}(z\ u)^{1-\alpha}\right] - \dot{z}/z \tag{D3.1}$$

with[8]

$$\Psi(t_Y) \equiv \frac{\phi}{\psi} = \frac{\alpha}{1-\alpha}\left(\frac{1-\alpha-(\varepsilon-\alpha)t_Y}{\alpha+(\varepsilon-\alpha)t_Y}\right) \qquad \text{and} \qquad \Phi(t_Y) \equiv \phi = \frac{\alpha(1-t_Y)}{\alpha+(\varepsilon-\alpha)t_Y}, \tag{T1}$$

and by one static relation:

$$l = \frac{\xi_l}{(1-t_Y)(1-\alpha)} \times \frac{x\ u}{\Psi(t_Y)^{\alpha}(z\ u)^{1-\alpha}} \tag{S1}$$

The balanced growth path equilibrium (hereafter BGP) is a stationary equilibrium where $u = u^\star$, $z = z^\star$, $x = x^\star$ are defined by $\dot{x} = \dot{z} = \dot{u} = 0$ and $l = l^\star$.

Proposition 1.

(i) Under the condition $B > \beta + \varrho$, there exists a unique balanced growth path equilibrium along which $u^\star \in](\beta+\varrho)/B, 1[$ solves $\Gamma^Y(u^\star, \Psi(t_Y)) = 0$ with

$$\Gamma^Y(u, \Psi(t_Y)) \equiv Bu - \beta + \frac{1-\alpha}{\alpha}\Psi(t_Y)\left(B\left(1-\bar{\mathcal{L}}^Y(u;\Psi(t_Y))\right) - \beta\right) - \frac{\beta(\beta+\varrho)}{Bu-\beta-\varrho}.$$

[8]Let us remark that when the abatement services are produced with the same technology than final output ($\varepsilon = \alpha$), we obtain $\phi = \psi = 1$.

(ii) The balanced growth path is saddle-path stable.

Proof. See appendix A. ■

Furthermore, along the BGP equilibrium the consumption to physical capital ratio is

$$x^\star = \frac{\beta(\beta+\varrho)}{Bu^\star - \beta - \varrho} > 0$$

the human capital to physical capital ratio is

$$z^\star = \frac{\Psi(t_Y)}{u^\star}\left[\frac{B(1-l^\star)-\beta}{\alpha(1-t_Y)}\right]^{1/(1-\alpha)} > 0$$

with leisure $l^\star$ defined as:

$$l^\star = \frac{1}{2}\left(\frac{B-\beta-\sqrt{(B-\beta)^2 - 4\xi_l \frac{\alpha B\,\beta(\beta+\varrho)\,u^\star}{(1-\alpha)\Psi(t_Y)(Bu^\star-\beta-\varrho)}}}{B}\right) > 0$$

Finally, the growth rate is:

$$g^\star = B(1-u^\star-l^\star) > 0$$

3 The environmental tax and the growth rate

In this section, we investigate the link between the environmental taxation and the rate of growth when pollution arises from production sectors. We will first take a look at the BGP equilibrium and then we will study the transition of the economy after an increase in the tax rate to highlight the economic mechanisms underlying the influence of the tax on the BGP equilibrium.

Proposition 2.

(i) When final output production and abatement production share the same technology and pollution arises from final output, tighter environmental tax has no impact on the long-run rate of growth in the case of finite lifetime, even if labor supply is endogenous.

(ii) Except in the case of infinite lifetime ($\beta = 0$) and exogenous labor supply ($\xi_l = 0$), if pollution arises from final output, tighter environmental tax promotes (respectively harms)

human capital accumulation in the long-run when the abatement sector is relatively more (resp. less) intensive in human capital than the final output sector. When lifetime is infinite and labor supply is exogenous, the environmental tax does not affect the long-run accumulation of human capital whatever the factor intensities.

Proof. See Appendix A. ■

Point *(i)* of proposition 2 states that the conventional findings *the environmental tax does not affect education in the long-run when labor supply is exogenous and lifetime is infinite* may be generalized to the finite lifetime case. It is in opposition with the result found by Pautrel (2011) when the source of pollution is physical capital. Furthermore, Proposition 2 states that the "*labor supply mechanism*" found by Hettich (1998) with infinite lifetime does not operate anymore when finite lifetime is taken into account.

To understand that result, we examine the influence of a tightening environmental tax during the transition of the economy towards the new BGP equilibrium. Because studying analytically the transition is cumbersome, we perform a numerical analysis using the Time-Elimination Method (see Mulligan and Sala-i Martin, 1991, 1993). We calibrate the model to obtain realistic values of the growth rate of GDP and the probability of death for the US economy. From the *World Development Indicators 2005* by the World Bank, life expectancy was 77.4 years in 2003 and the growth rate was 3.3% during the period 1990-2002. Since the expected lifetime is the reverse of the probability of death per unit of time β, we want β to be close to $1/77.4 = 0.0128$. We adjust other variables to obtain such values for our benchmark case. We investigate the influence of the technology during the transition by considering two polar cases : $\varepsilon = 0$ (to study $\varepsilon < \alpha$) and $\varepsilon = 1$ (to study $\varepsilon > \alpha$). In the first case (*resp. the second case*), the abatement sector only uses human capital (*resp. physical capital*), that is $\phi = 1$ (*resp.* $\psi = 1$).[9]

Table 1 gives the benchmark value of parameters and Table 2 summarizes the exercise of comparative statics. Graph 1 to 3 (at the end of the article) draw the temporal evolution of the main variables towards the new steady-state when an unanticipated increase in the environmental tax is implemented by the government, respectively for $\alpha = \varepsilon = 1/3$, $\alpha > \varepsilon = 0$ and $\alpha < \varepsilon = 1$.

[9]For $\varepsilon = 0$, we obtain $\partial\phi/\partial t_Y = 0$. Otherwise $\partial\phi/\partial t_Y < 0$. For $\varepsilon = 1$, we obtain $\partial\psi/\partial t_Y = 0$. Otherwise $\partial\psi/\partial t_Y < 0$. Furthermore $\partial\Psi(t_Y)/\partial t_Y \geq 0$ for $\alpha \geq \varepsilon$.

Table 1: Benchmark value of parameters

α	ϱ	B	β	ε
1/3	0.025	0.085	0.0128	1/3

Table 2: The increase in the environmental tax along the BGP

ε	1/3		0		1	
t_Y	0.01	0.1	0.01	0.1	0.01	0.1
$g^\star(\%)$	3.667	3.667	3.674	3.732	3.653	3.520
$u^\star$	0.481	0.481	0.480	0.476	0.482	0.490
$l^\star$	0.088	0.088	0.087	0.085	0.089	0.096
$x^\star$	0.158	0.158	0.160	0.179	0.154	0.125
$z^\star$	0.181	0.209	0.184	0.247	0.175	0.156
$r^\star + \beta$	0.0775	0.0775	0.0776	0.0778	0.0775	0.0769

Graph 1 helps to illustrate the economic mechanisms underlying the Proposition 2*(i)*. At the impact, the tighter environmental policy brings down the returns to physical capital (r) and the wage rate (w) such that the ratio w/r rises (see Graph 1.i & 1.ii). The decrease in the wage rate reduces the returns to education but globally (taking into account the effect of leisure) the returns to education is higher than the returns to physical capital investment and the difference becomes positive (see Graph 1.iii). Agents allocate a part of their resources from final output to human capital accumulation: u jumps downward (see Graph 1.iv). Due to the combined fall of u and w, the present value of lifetime earnings drops and agents arise their saving: consumption falls at impact and therefore the aggregate consumption to physical capital ratio (x) jumps downward because K is predetermined (see Graph 1.v). Because the fall in w is higher than the fall in the aggregate consumption, leisure time chosen by individuals jumps at the impact (see Graph 1.vi) and reduces the rewards to education but globally the returns to education remains higher than the returns to physical capital (see Graph 1.iii). The greater investment in education leads to a substitution between physical capital and human capital: z rises continuously during the transition (recall that physical capital investment requires polluting final output) as shown in Graph 1.vii. The rise in z increases the interest rate and therefore reduces the gap between the two returns to capital: during the transition u rises and that contributes (with the rise of z) to diminish more the wage rate w (see Graph 1.ii). Furthermore, when the interest rate goes back to its initial value, the ratio aggregate consumption to physical capital x backs to its initial value. Therefore, despite the generational turnover effect that disconnects the aggregate consumption growth to the interest rate (the term $\beta(\varrho+\beta)x^{-1}$ in equation 14), the aggregate consumption growth backs to its initial value and the physical capital rate of growth as well (see Graph 1.viii & 1.ix). The human capital accumulation is then back to its initial level (see Graph 1.x), that is to say u, l and all variables back to their initial value except the wage rate and the aggregate human capital to aggregate physical capital ratio z (see Graph 1.ii & 1.vii). It is higher because of the substitution between the stocks of capital. Therefore a tighter environmental tax has no impact on the long-run growth rate (see Graph 1.xi).

Point *(ii)* of proposition 2 states that assuming a different technology for final output production and abatement services production leads to two important insights. First, the environmental tax influences the BGP rate of growth. That result challenges the conventional result that the environmental tax does not affect the growth rate when pollution arises from final output, in the Lucas (1988) settings, originally demonstrated by Gradus and Smulders (1993) and extended to the case of endogenous labor supply by Hettich (1998). Second, according to the relative factor intensity of each sector, the influence of the environmental tax may be positive or negative. When the abatement sector is more (*respectively less*) intensive in human capital than the final output sector, that is $\alpha > \varepsilon$ (*resp.* $\alpha < \varepsilon$), the environmental tax enhances (*resp. reduces*) the BGP rate of growth. When the same technology is used for final production and abatement production ($\alpha = \varepsilon$), the environmental tax does not affect BGP growth.

That result may be explained as follows. At the impact, the tighter environmental tax has two effects: *(i)* it reduces the rewards to physical capital (r) and to human capital (w), *(ii)* it leads to a reallocation of factors between the final output sector and the abatement sector when the technology used in both sectors are different. That second impact is the source of the positive (*resp. negative*) influence of the environmental policy on long-run growth when the abatement sector is more (*resp. less*) intensive in human capital than the final output sector, that is when $\alpha > \varepsilon$ (*resp.* $\alpha < \varepsilon$).

When the abatement sector is relatively intensive in human capital, the increase in the environmental tax leads to a rise in abatement services production, requiring more human capital. Because human capital is freed from the output sector relatively more intensive in physical capital, there is a higher pressure on the human capital rewards: w does not fall at the impact as it did with the same factor intensity in both sectors (compare Graph 2.ii with Graph 1.ii). Conversely r falls more because more physical capital is relatively released for one input of human capital reallocated from the output sector to the abatement sector (compare Graph 2.i with Graph 1.i). As a consequence, the drop of the aggregate consumption to physical capital ratio x at the impact is lowered (see Graph 2.v) and the drop of u is higher because the gap between returns is higher (see Graph 2.iii). Therefore the jump of l is reduced (see Graph 2.vi). Adjustment mechanisms towards the new balanced-growth path equilibrium are similar to the case $\alpha = \varepsilon$: x and z rise, l falls. Nevertheless the amplitude of variations is magnified and x and l go respectively above and below their initial values (see Graph 2.v & 2.vi). The human capital accumulation is boosted while the accumulation of physical capital drops more (see Graph 2.x & 2.viii) so that the equalization of returns is made for a value of u lower than its initial value as shown in Graph 2.iv (it is also due to a greater increase of z).

When the abatement sector is relatively intensive in physical capital ($\alpha < \varepsilon$), mechanisms are modified. Because the tighter environmental tax rises the production in the abatement sector, due to the difference in factor intensities between sectors, physical capital is relatively scarce and therefore despite the negative impact of the tighter environmental tax, the interest rate jumps at the impact (see Graph 3.i). On the other hand, the freed human capital in the output sector reinforces the fall-off in the wage rate due to the tighter environmental tax (see Graph 3.ii). As a result, the difference between the returns to human capital and the physical capital becomes negative at the impact (see Graph 3.iii) and agents reallocate their human capital towards output production: u rises at the impact (see Graph 3.iv). In the same time, the reduction in the wage rate diminishes the discounted value of earnings (human wealth) while the interest payments on non-human wealth rises: agents increase their saving and consumption jumps downward the impact: x falls because K is pre-determined (see Graph 3.v). Due to the increase in u, the human capital accumulation falls at the impact while the higher interest rate leads to an increase in the investment in physical capital (see Graph 3.x & 3.viii). That reinforces the drop in x, favoring the increase in l at the impact and during the transition (see Graph 3.vi). Finally, the aggregate human capital to aggregate physical capital ratio z falls towards a new long-run value lower than the initial value (see Graph 3.vii). That fall leads the interest rate to be lower in the long-run, that is when the gap between returns vanished (see Graph 3.i). That leads to a new steady-state rate of growth lower than its initial value (see Graph 3.xi). The tighter environmental reduces the long-run growth rate.

4 Discussion

In the previous section we demonstrated that the environmental policy affects long-run growth in a Lucas (1988)' model even if the source of pollution is final output, when production technology in the abatement sector differs from the production technology in the final output sector. We also demonstrated that this influence of the environmental policy on growth could be either positive or negative. The purpose of this section is to investigate whether the conventional positive impact of the environmental tax on growth with physical capital as the source of pollution remains valid when abatement technology is modified.

We assume that physical capital is the source of pollution and therefore physical capital income is taxed at a rate t_k. Therefore, the after-tax real interest rate is $r_n = (1 - t_k)\ r$ and equation (10) becomes:

$$t_k r K = P_D D \tag{15}$$

Furthermore, equations (12) and (13) become:

$$r = \alpha \frac{G}{\phi K} = \varepsilon \frac{P_D D}{(1-\phi)K} \tag{16a}$$

$$w = (1-\alpha)\frac{G}{\psi H_p} = (1-\varepsilon)\frac{P_D D}{(1-\psi)H_p} \tag{16b}$$

Using (15) and dividing member by member equations (16a) and (16b), we obtain

$$\frac{1-\psi}{\psi} = \left(\frac{1-\varepsilon}{\varepsilon}\right)\left(\frac{\alpha}{1-\alpha}\right)\frac{1-\phi}{\phi} \qquad \text{that is} \qquad \psi = \frac{(1-\alpha)\varepsilon}{(\varepsilon-\alpha)\phi + \alpha(1-\varepsilon)}\phi$$

From (15), we also have $r = \varepsilon \frac{t_k r K}{(1-\phi)K}$, that is

$$\phi = 1 - \varepsilon t_k > 0$$

Finally, from (16a), $rt_kK = \left(\frac{\alpha}{\varepsilon}\right)\left(\frac{1}{\phi} - 1\right)G$. Because final output is used for consumption and physical capital investment, the market clearing condition yields:

$$\left(1 + \left(\frac{\alpha}{\varepsilon}\right)\left(\frac{1}{\phi} - 1\right)\right)G = C + \dot{K}$$

Therefore, the dynamical system is

$$\dot{x}/x = [\alpha(1-t_k) - 1 - (\alpha - \varepsilon)t_k]\,\Upsilon(t_k)^{\alpha-1}(z\ u)^{1-\alpha} - \varrho - \beta(\beta+\varrho)x^{-1} + x \tag{D2.1}$$

$$\dot{z}/z = B(1-u-l) - (1 + (\alpha-\varepsilon)t_k)\Upsilon(t_k)^{\alpha-1}(z\ u)^{1-\alpha} + x \tag{D2.2}$$

$$\dot{u}/u = \alpha^{-1}\left[B(1-l) - \beta - \alpha(1-t_k)(z\ u)^{1-\alpha}\right] - \dot{z}/z \tag{D3.2}$$

where $\Upsilon(t_k) \equiv \frac{\phi}{\psi} = 1 + \frac{\alpha-\varepsilon}{1-\alpha}t_k$ (with $\Upsilon_{t_k}(t_k) \geq 0 \ \ if \ \ \alpha \geq \varepsilon$),[10] and

$$l = \frac{\xi_l}{(1-\alpha)\Upsilon(t_k)^{\alpha}} \times \frac{x\ u}{(z\ u)^{1-\alpha}} \tag{S2}$$

Solving the dynamic system gives the following proposition:

Proposition 3. *If pollution arises from the stock of physical capital, tighter environmental tax promotes long-run human capital accumulation whatever the relative factor intensity in the abatement sector and the final output sector.*

Proof. See Appendix B. ■

Proposition 3 states that abatement technology does not impact the growth-effect of the environmental policy when physical capital is the source of pollution. That result may be explained in the case $\alpha < \varepsilon$ (the abatement sector is relatively more intensive in physical capital) as follows. When physical capital is taxed, the relative factor reward r/w drop more (with respect to the case where output is taxed). Therefore, the time allocated to production (u) diminishes more and the fall of the interest rate is so high that even if physical capital is relatively scarce in output production (because abatement production is relatively more intensive in physical capital), the rising force due to that scarcity is not enough to make the interest rate jumping upward at the impact. Therefore, the overall adjustment mechanisms remain the same whatever the relative factorial intensity in final output and abatement sectors.

[10] Let us remark that when the abatement services are produced with the same technology than final output ($\varepsilon = \alpha$), we obtain $\phi = \psi = 1 - \varepsilon t_k$. Therefore, $G = (1-\varepsilon t_k)K^{\alpha}H_p^{1-\alpha}$, $P_D D = \varepsilon t_k\ K^{\alpha}H_p^{1-\alpha}$ and $Y = K^{\alpha}H_p^{1-\alpha}$.

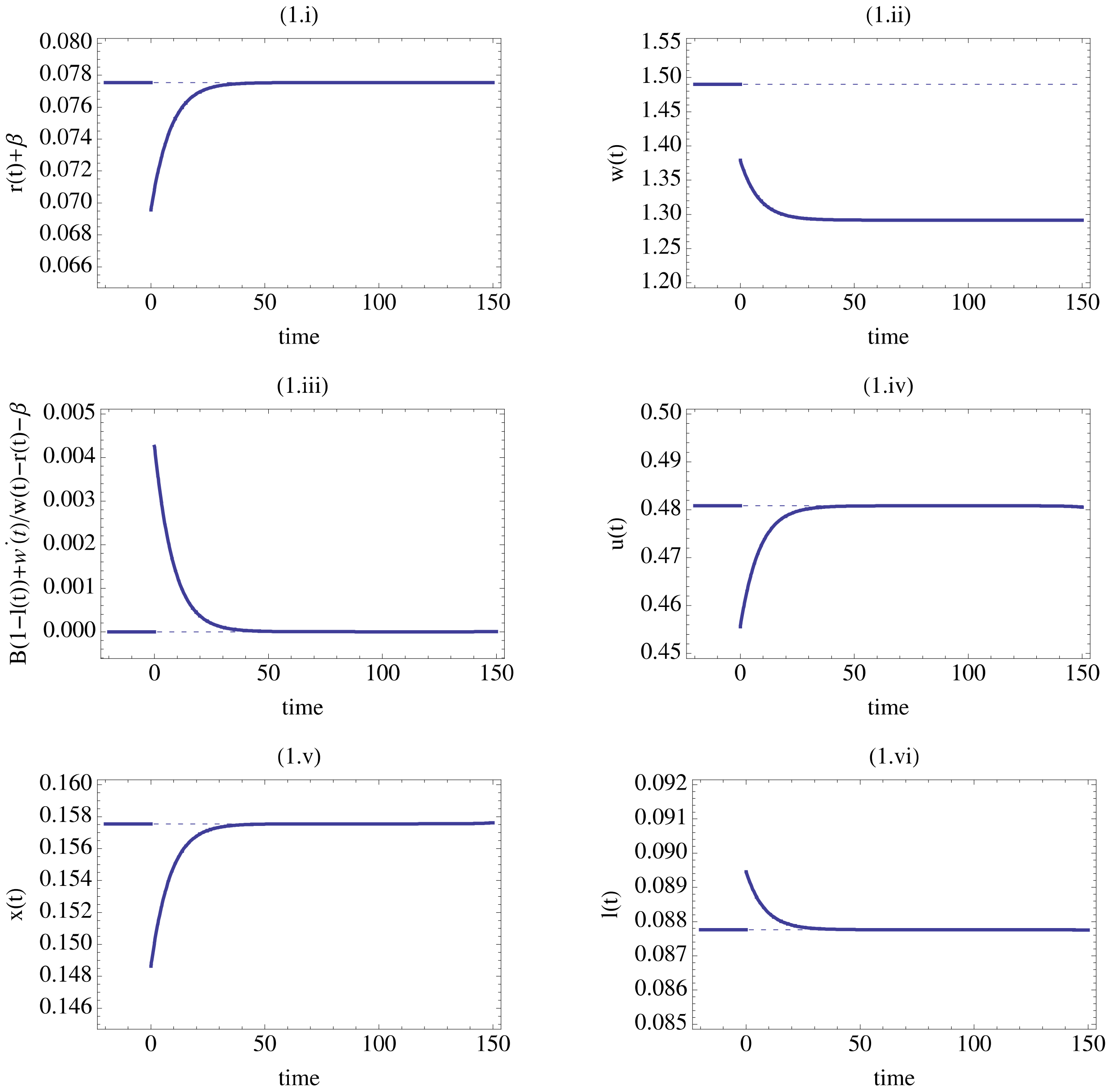
5 Conclusion

The aim of the article is to re-examine the growth-effects of a tighter environmental tax when the source of pollution is final output and human capital accumulation is the engine of growth. Compared with previous articles, we take into account finitely-lived agents and we relax the basic assumption that technologies of production are similar in final output and abatement sectors.

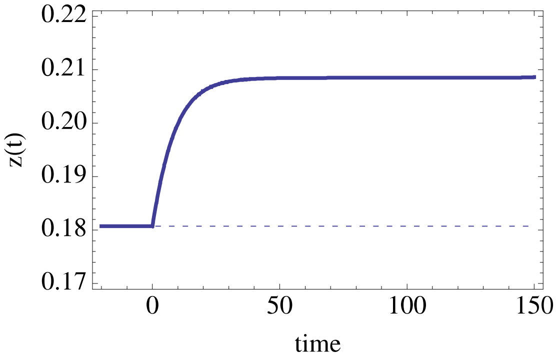
We demonstrate that the results found by Gradus and Smulders (1993) and Hettich (1998) can be generalized to finite lifetime when production technology across sectors is similar: the environmental tax has no growth-effect if pollution arises from final output. Nevertheless, we demonstrate that the environmental tax will boost (*respectively harm*) long-run human capital accumulation when the abatement sector is relatively more intensive in human capital (*resp. physical capital*) than the output sector. That result always holds for finite lifetime but for infinite lifetime it holds only when labor supply is endogenous. Therefore, the findings of Gradus and Smulders (1993) in the case of exogenous labor supply and a similar technology in abatement and output sectors remain valid when technologies between sectors differ. Our final contribution is to demonstrate that the environmental policy always promote long-run human capital accumulation whatever the technologies used in final output sector and in the abatement sector, when pollutant emissions originate from physical capital.

Thus our article gives new important insights to understand how environmental tax impacts long-run growth when education is the channel of transmission. Highlighting that the technology used in the abatement sector determines the existence and the direction of the growth-effect, it calls for more interest in the modeling of the abatement "*side*" of the growth model with environment.

Graph 1. $\alpha = \varepsilon$ **(to be continued...)**



(1.vii)



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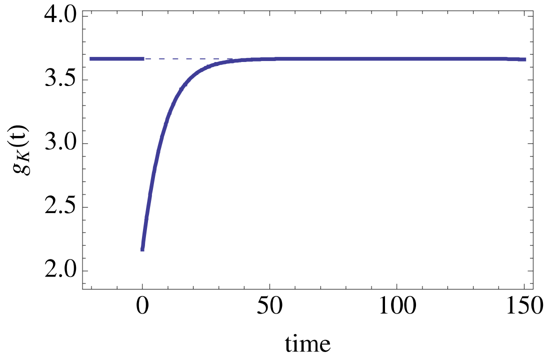

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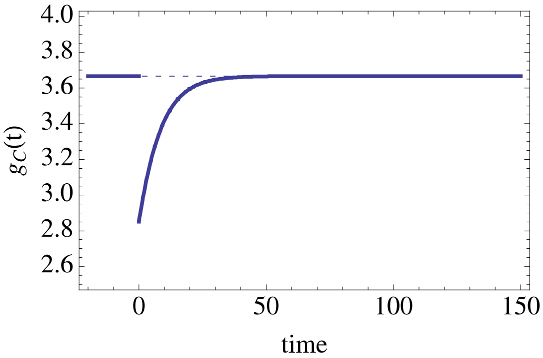

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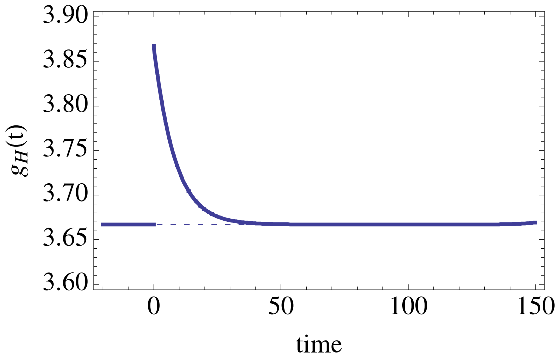

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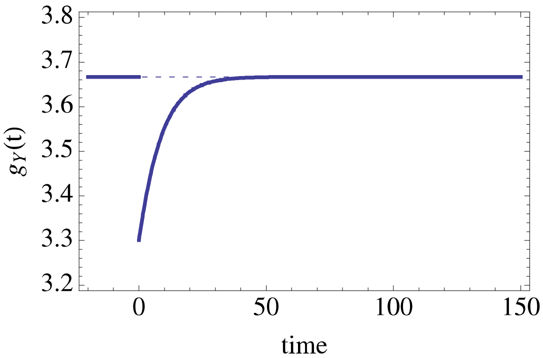
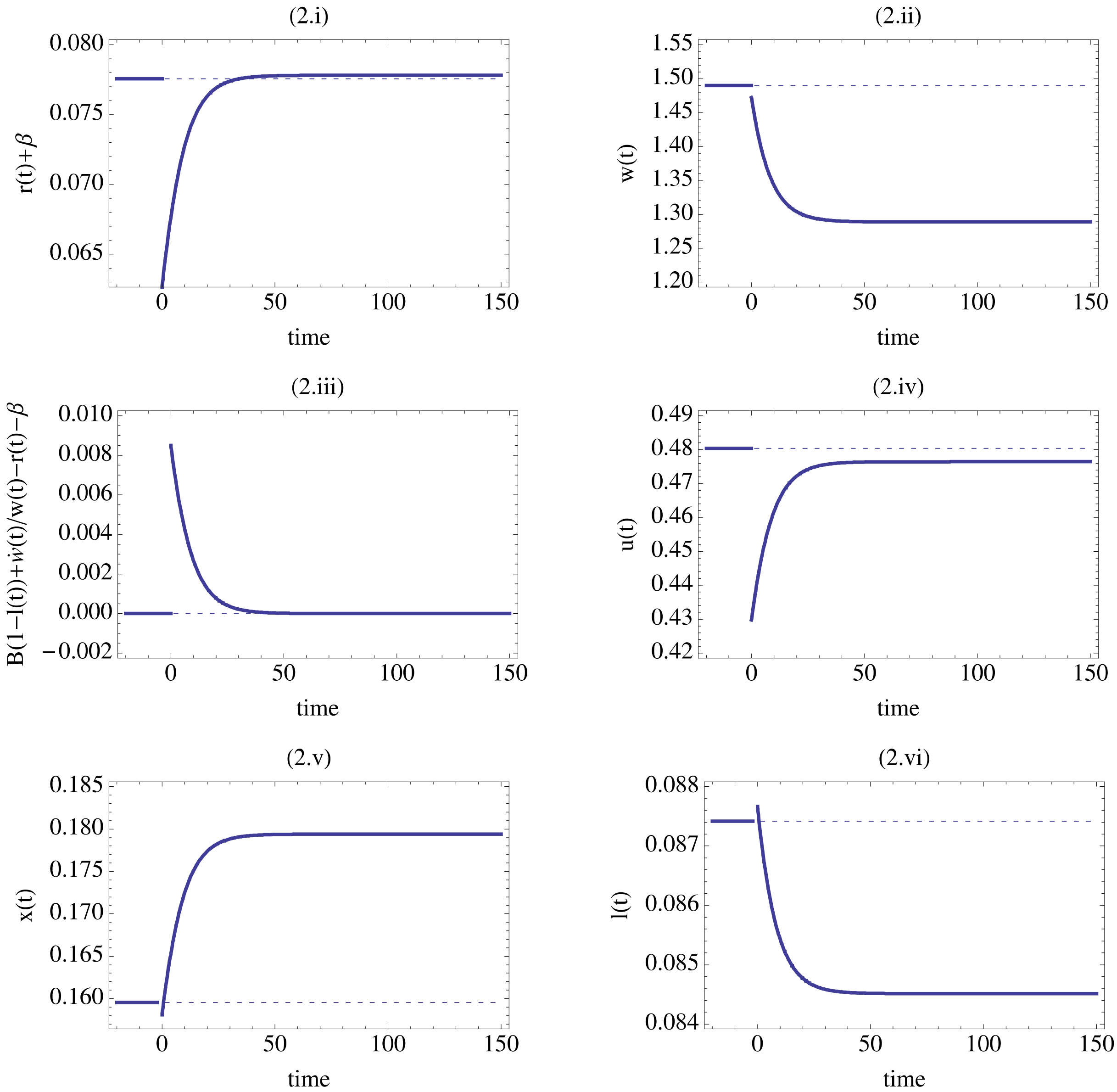
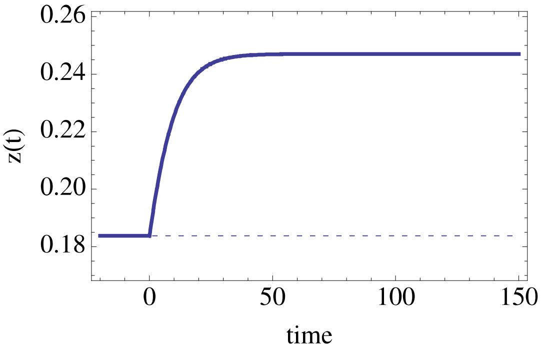


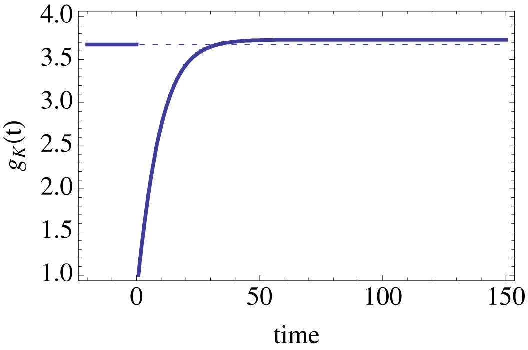
Graph 2. $\alpha > \varepsilon$ **(to be continued...)**



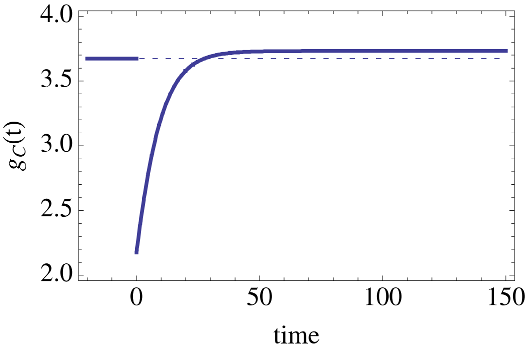
(2.vii)



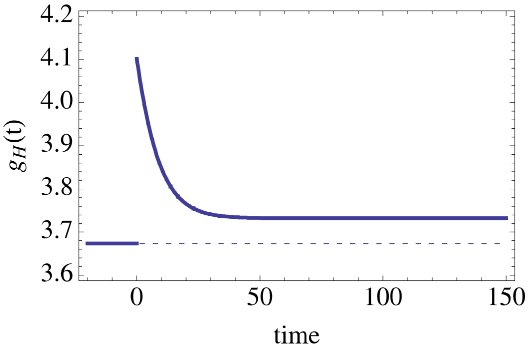
(2.viii)



(2.ix)



(2.x)



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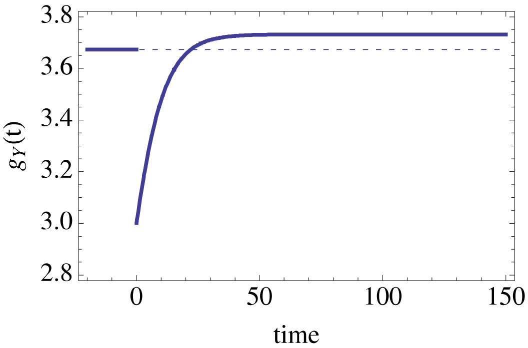
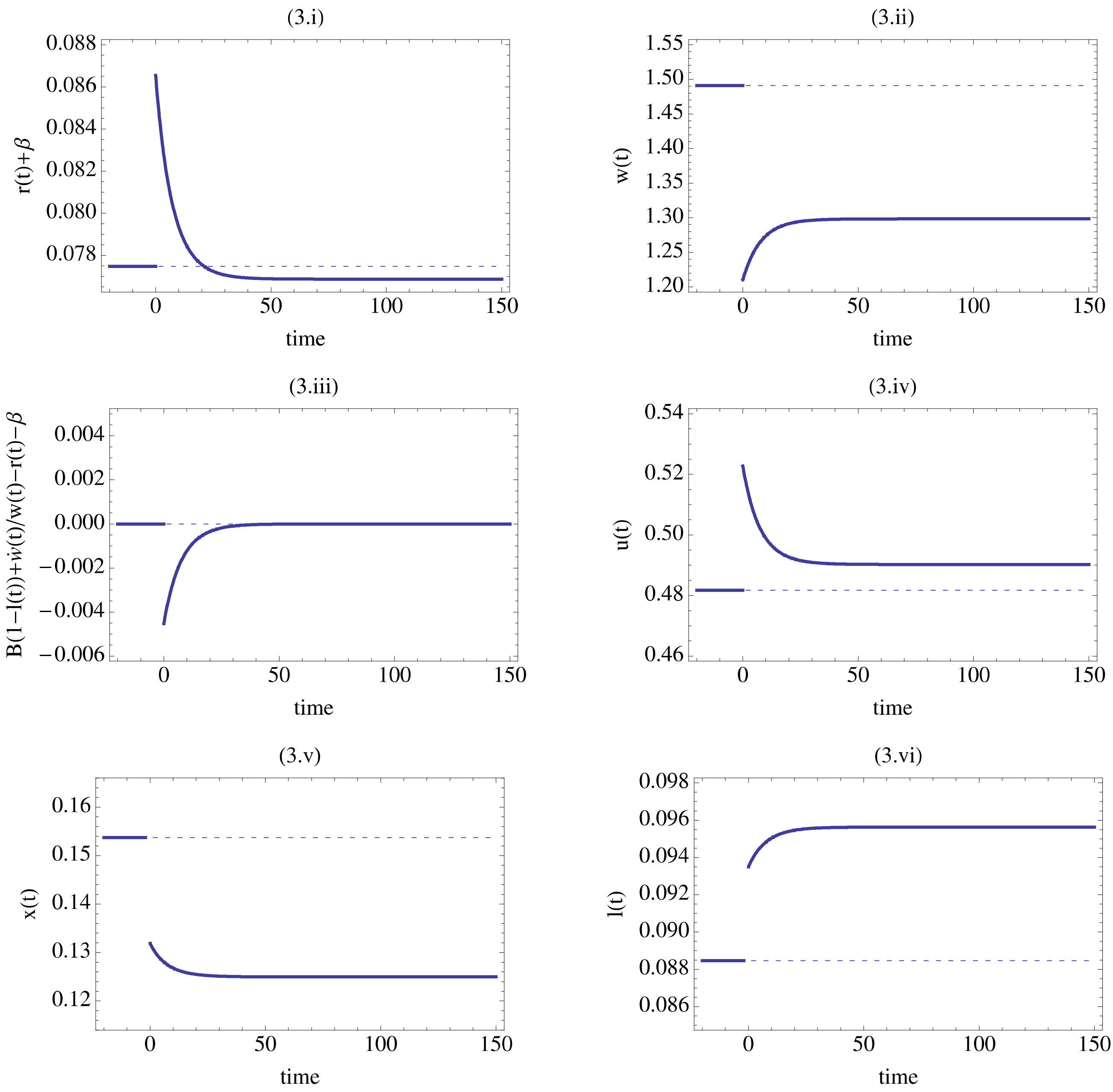
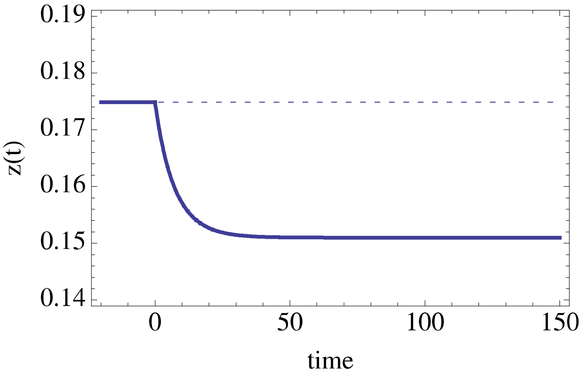


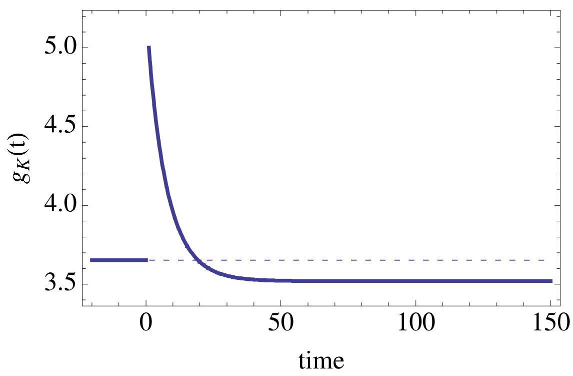
Graph 3. $\alpha < \varepsilon$ **(to be continued...)**



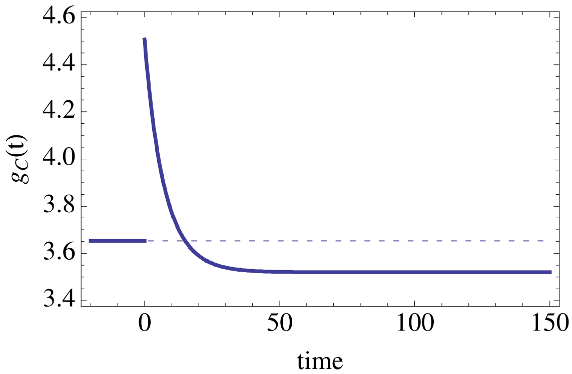
(3.vii)



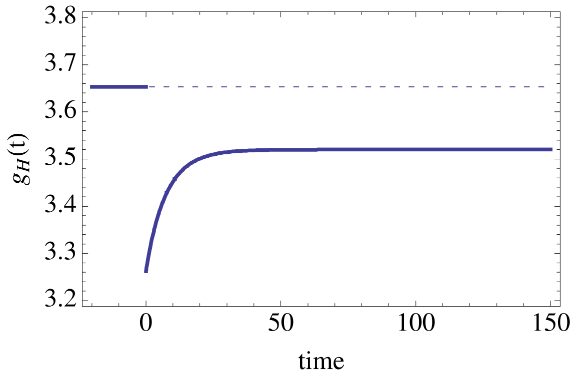
(3.viii)



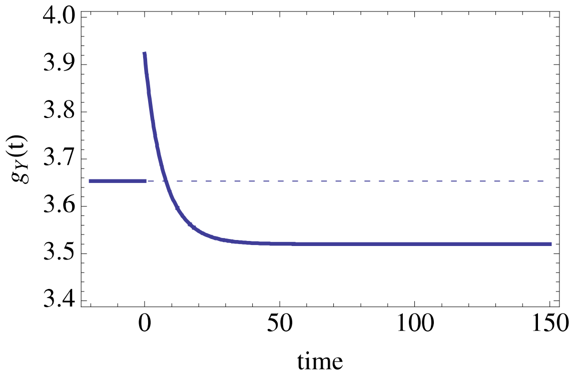
(3.ix)



(3.x)



(3.xi)



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A Existence, unicity and stability of the balanced growth path equilibrium

A.1 The case $\beta > 0$ and $\xi_l > 0$

From (D3.1), $\dot{u} = 0$, we obtain

$$(z^\star\ u^\star)^{1-\alpha} = \frac{\Psi(t_Y)^{1-\alpha}}{\alpha(1-t_Y)}\,[B(1-l^\star)-\beta] \tag{A.1}$$

Because $z^\star\ u^\star > 0$, it implies

$$B(1-l^\star)-\beta > 0 \qquad \Rightarrow \qquad B > \beta \tag{cond1}$$

Equation (S1) also defines a relation between $l^\star$ and $z^\star u^\star$:

$$l^\star = \frac{\xi_l\ x^\star\ u^\star}{(1-\alpha)(1-t_Y)\Psi(t_Y)^\alpha(z^\star u^\star)^{1-\alpha}}$$

such that using (A.1), we obtain

$$l^\star = \frac{\xi_l\alpha}{(1-\alpha)\Psi(t_Y)} \times \frac{x^\star\ u^\star}{B(1-l^\star)-\beta} \tag{A.2}$$

Therefore (A.2) gives the expression of $l^\star \in [0,1[$:[11]

$$l^\star = \tilde{\mathcal{L}}^Y(x^\star,u^\star;\Psi(t_Y)) \equiv \frac{1}{2}\left(\frac{B-\beta-\sqrt{(B-\beta)^2-4\frac{\xi_l\alpha}{(1-\alpha)\Psi(t_Y)}B\ x^\star\ u^\star}}{B}\right)$$

with $\tilde{\mathcal{L}}^Y_{x^\star}(x^\star,u^\star;\Psi(t_Y)) > 0$, $\tilde{\mathcal{L}}^Y_{u^\star}(x^\star,u^\star;\Psi(t_Y)) > 0$, $\tilde{\mathcal{L}}^Y_{\Psi(t_Y)}(x^\star,u^\star;\Psi(t_Y)) < 0$. Furthermore, from (D1.1) and (D2.1), $\dot{x}-\dot{z} = 0$ at the steady-state gives (with A.1):

$$x^\star = \frac{\beta(\beta+\varrho)}{Bu^\star-\beta-\varrho} \tag{A.3}$$

Because $x^\star > 0$, it is required that

$$u^\star > \frac{\beta+\varrho}{B} \qquad \text{with } \ B > \beta+\varrho \tag{cond2}$$

[11] Equation (A.2) is a quadratic equation of $l^\star$ that have one positive and one negative solution. Only the positive one is interesting for us.

Therefore, $x^\star\ u^\star$ is an increasing function of $u^\star$ and defining $\chi(u^\star) \equiv x^\star\ u^\star$, we can express the endogenous labor supply at steady-state as:

$$l^\star = \bar{\mathcal{L}}^Y(u^\star;\Psi(t_Y)) \equiv \frac{1}{2}\left(\frac{B-\beta-\sqrt{(B-\beta)^2-4\frac{\xi_l\alpha}{(1-\alpha)\Psi(t_Y)}B\ \chi(u^\star)}}{B}\right) \tag{A.4}$$

with $\bar{\mathcal{L}}^Y_{u^\star}(u^\star;\Psi(t_Y)) > 0$, $\bar{\mathcal{L}}^Y_{\Psi(t_Y)}(u^\star;\Psi(t_Y)) < 0$. $\dot{z} = 0$ leads to, using (D2.1), (A.1) and (T1):

$$x^\star = Bu^\star - \left(1+\frac{1-\alpha}{\alpha}\Psi(t_Y)\right)\beta + \frac{1-\alpha}{\alpha}\Psi(t_Y)B\left[1-\bar{\mathcal{L}}^Y(u^\star;\Psi(t_Y))\right] \tag{A.5}$$

Equating (A.3) and (A.5) enables to express $u^\star$ as the solution of

$$\Gamma^Y(u,\Psi(t_Y)) = 0$$

where

$$\Gamma^Y(u,\Psi(t_Y)) \equiv Bu-\beta+\frac{1-\alpha}{\alpha}\Psi(t_Y)\left(B\left(1-\bar{\mathcal{L}}^Y(u;\Psi(t_Y))\right)-\beta\right)-\frac{\beta(\beta+\varrho)}{Bu-\beta-\varrho}$$

$$\text{with} \quad \Gamma^Y_u(u,\Psi(t_Y)) > 0 \quad \text{and} \quad \Gamma^Y_{\Psi(t_Y)}(u,\Psi(t_Y)) > 0$$

because

$$\Gamma^Y_u(u,\Psi(t_Y)) = B + \frac{B\beta(\beta+\varrho)}{(Bu-\beta-\varrho)^2} + \xi_l\frac{\frac{B\beta(\beta+\varrho)^2}{(Bu-\beta-\varrho)^2}}{\sqrt{(B-\beta)^2-\frac{4\alpha\xi_l}{(1-\alpha)\Psi(t_Y)}B\ \chi(u^\star)}} > 0$$

From (cond2), $u^\star \in](\beta+\varrho)/B, 1[$ with $B > \beta+\varrho$. We have $\lim_{u\to(\beta+\varrho)/B} = -\infty$ and $\lim_{u\to 1} > 0$ because $B-\beta-\varrho > \beta(\beta+\varrho)$ (sufficient condition). Therefore, $u^\star \in](\beta+\varrho)/B, 1[$ is unique.

From the theorem of the implicit function we have

$$u^\star = \bar{\mathcal{U}}^Y(\Psi(t_Y)) \qquad \text{with} \qquad \bar{\mathcal{U}}^Y_{\Psi(t_Y)}(\Psi(t_Y)) < 0$$

From (T1), we have $\Psi_{t_Y}(t_Y) \gtreqless 0$ when $\alpha \gtreqless \varepsilon$, therefore it comes

$$u^\star = \mathcal{U}^Y(t_Y) \qquad \text{with} \qquad \mathcal{U}^Y_{t_Y}(t_Y) \leq 0 \ \text{ when } \ \alpha \geq \varepsilon$$

From (A.4), we obtain that

$$l^\star = \mathcal{L}^Y(t_Y) \qquad \text{with} \ \ \mathcal{L}^Y_{t_Y}(t_Y) \leq 0 \ \text{ when } \ \alpha \geq \varepsilon$$

As a result

$$g^\star = B(1-\mathcal{U}^Y(t_Y)-\mathcal{L}^Y(t_Y)) \qquad \text{with} \ \ g^\star_{t_Y} \geq 0 \ \text{ when } \ \alpha \geq \varepsilon$$

A.2 The case $\beta = 0$

When lifetime is infinite, $\beta = 0$ and the five equations (D1.1-D3.1, T1, S1) becomes:

$$\dot{x}/x = [\alpha(1-t_Y) - \Phi(t_Y)]\, \Psi(t_Y)^{\alpha-1}(z\ u)^{1-\alpha} - \varrho + x \tag{D1.1a}$$

$$\dot{z}/z = B(1-u-l) - \Phi(t_Y)\Psi(t_Y)^{\alpha-1}(z\ u)^{1-\alpha} + x \tag{D2}$$

$$\dot{u}/u = \alpha^{-1}\left[B(1-l) - (1-\tau)\alpha\Psi(t_Y)^{\alpha-1}(z\ u)^{1-\alpha}\right] - \dot{z}/z \tag{D3.1a}$$

To obtain the expression of the BGP rate of growth, just recall that $g^\star = r^\star - \varrho$ where $r^\star$ is the interest rate along the BGP defined as $r^\star = (1-\tau)\alpha\Psi(t_Y)^{\alpha-1}(z^\star\ u^\star)^{1-\alpha}$. Therefore, along the BGP $\dot{u} = 0$, implies that

$$r^\star = B(1-l^\star) \qquad\qquad \Rightarrow \qquad\qquad l^\star = 1 - \frac{r^\star}{B}$$

Furthermore, from (D1.1a),

$$x^\star = \varrho - \left[1 - \frac{\Phi(t_Y)}{\alpha(1-t_Y)}\right]\ r^\star = \varrho + \Psi(t_Y) r^\star$$

and $\dot{x} - \dot{z} = 0$ leads to

$$u^\star = \frac{\varrho}{B}$$

Therefore equation (S1) gives the implicit expression of $r^\star$:

$$1 - \frac{r^\star}{B} = \xi_l \frac{[\varrho + \Psi(t_Y) r^\star]\ \ \varrho/B}{\left(\frac{1-\alpha}{\alpha}\right)\Psi(t_Y)\ r^\star}$$

that is

$$B - r^\star = \frac{\xi_l}{\left(\frac{1-\alpha}{\alpha}\right)\ \Psi(t_Y)}\left[\frac{\varrho}{r^\star} + \Psi(t_Y)\right]\ \varrho$$

whose the unique positive solution is:

$$r^\star = \frac{(1-\alpha)B - \xi_l\alpha\varrho + \sqrt{((1-\alpha)B - \xi_l\alpha\varrho)^2 - 4\xi_l\alpha(1-\alpha)\varrho^2/\Psi(t_Y)}}{2(1-\alpha)} > 0$$

It is straightforward that $\partial r^\star/\partial\Psi(t_Y) > 0$, $\forall \xi_l > 0$. Because $\partial\Psi(t_Y)/\partial t_Y \geq 0$ if and only if $\alpha \geq \varepsilon$ then $\forall \xi_l > 0$, $\partial r^\star/\partial t_Y \geq 0$ if and only if $\alpha \geq \varepsilon$. When $\xi_l = 0$, $r^\star = B$ independent from t_Y.

B The balanced-growth path equilibrium when K is the source of pollution

From (D3.2), $\dot{u} = 0$, we obtain

$$(z^\star \ u^\star)^{1-\alpha} = \frac{\Upsilon(t_k)^{1-\alpha}}{\alpha(1-t_k)} \left[B(1-l^\star) - \beta\right] \tag{B.1}$$

with $\Upsilon(t_k) \equiv 1 + \frac{\alpha-\varepsilon}{1-\alpha} t_k$. Because $z^\star \ u^\star > 0$, condition (cond1) $B > \beta$ always holds. Equation (S1) also defines a relation between $l^\star$ and $z^\star u^\star$:

$$l^\star = \frac{\xi_l \ x^\star \ u^\star}{(1-\alpha)\,\Upsilon(t_k)^\alpha (z^\star u^\star)^{1-\alpha}}$$

such that using (B.1), we obtain

$$l^\star = \frac{\xi_l \alpha(1-t_k)}{(1-\alpha)\Upsilon(t_k)} \times \frac{x^\star \ u^\star}{B(1-l^\star) - \beta} \tag{B.2}$$

Note that $\frac{1-t_k}{\Upsilon(t_k)} = \frac{(1-\alpha)(1-t_k)}{1-\alpha+(\alpha-\varepsilon)t_k}$ and $\partial \frac{1-t_k}{\Upsilon(t_k)} / \partial t_k \leq 0$.[12] Therefore (B.2) gives the expression of $l^\star \in [0,1[$:[13]

$$l^\star = \tilde{\mathcal{L}}^K(x^\star, u^\star; t_k) \equiv \frac{1}{2} \left(\frac{B - \beta - \sqrt{(B-\beta)^2 - 4\frac{\alpha \xi_l (1-t_k)}{(1-\alpha)\Upsilon(t_k)} B \ x^\star \ u^\star}}{B} \right)$$

with $\tilde{\mathcal{L}}^K_{x^\star}(x^\star, u^\star; t_k) > 0$, $\tilde{\mathcal{L}}^K_{u^\star}(x^\star, u^\star; t_k) > 0$, $\tilde{\mathcal{L}}^K_{t_k}(x^\star, u^\star; t_k) \leq 0$. Furthermore, from (D2.1) and (D2.2), $\dot{x} - \dot{z} = 0$ at the steady-state gives (with B.1):

$$x^\star = \frac{\beta(\beta+\varrho)}{Bu^\star - \beta - \varrho} \tag{B.3}$$

Therefore, $x^\star \ u^\star$ is an increasing function of $u^\star$ and defining $\chi(u^\star) \equiv x^\star \ u^\star$, we can express the endogenous labor supply at steady-state as:

$$l^\star = \bar{\mathcal{L}}^K(u^\star; t_k) \equiv \frac{1}{2} \left(\frac{B - \beta - \sqrt{(B-\beta)^2 - 4\frac{\alpha \xi_l (1-t_k)}{(1-\alpha)\Upsilon(t_k)} B \ \chi(u^\star)}}{B} \right) \tag{B.4}$$

[12] It directly comes from the fact $\partial \frac{1-t_k}{\Upsilon(t_k)} / \partial t_k = -\frac{(1-\alpha)(1-\varepsilon)}{[1-\alpha+(\alpha-\varepsilon)t_k]^2} \leq 0$, $\forall (\alpha, \varepsilon) \in (0,1)$.

[13] Equation (B.2) is a quadratic equation of $l^\star$ that have one positive and one negative solution. Only the positive one is interesting for us.

with $\bar{\mathcal{L}}^K_{u^\star}(u^\star; t_k) > 0$, $\bar{\mathcal{L}}^K_{t_k}(u^\star; t_k) \leq 0$. $\dot{z} = 0$ leads to, using (D2.2) and (B.1):

$$x^\star = Bu^\star - \beta + \left(\frac{1 + (\alpha - \varepsilon)t_k}{\alpha(1 - t_k)} - 1 \right) \left[B \left(1 - \bar{\mathcal{L}}^K(u^\star; t_k) \right) - \beta \right] \tag{B.5}$$

Equating (B.3) and (B.5) enables to express $u^\star$ as the solution of

$$\Gamma^K(u, t_k) = 0$$

where

$$\Gamma^K(u, t_k) \equiv Bu^\star - \beta + \left(\frac{1 + (\alpha - \varepsilon)t_k}{\alpha(1 - t_k)} - 1 \right) \left[B \left(1 - \bar{\mathcal{L}}^K(u^\star; t_k) \right) - \beta \right] - \frac{\beta(\beta + \varrho)}{Bu - \beta - \varrho}$$

$$\text{with} \qquad \Gamma^K_u(u, t_k) > 0 \qquad \text{and} \qquad \Gamma^K_{t_k}(u, t_k) > 0, \;\; \forall\ (\alpha, \varepsilon) \in]0, 1[$$

because $\partial \left(\frac{1+(\alpha-\varepsilon)t_k}{\alpha(1-t_k)} - 1 \right) / \partial t_k = \frac{1+\alpha-\varepsilon}{\alpha(1-t_k)^2} > 0$. From (cond2), $u^\star \in](\beta + \varrho)/B, 1[$ with $B > \beta + \varrho$. We have $\lim_{u \to (\beta+\varrho)/B} = -\infty$ and $\lim_{u \to 1} > 0$ because $B - \beta - \varrho > \beta(\beta + \varrho)$ (sufficient condition). Therefore, $u^\star \in](\beta + \varrho)/B, 1[$ is unique.

From the theorem of the implicit function we have

$$u^\star = \mathcal{U}^K(t_k) \qquad \text{with} \qquad \mathcal{U}^K_{t_k}(t_k) < 0$$

From (B.4), we obtain that

$$l^\star = \mathcal{L}^K(t_k) \qquad \text{with} \;\; \mathcal{L}^K_{t_k}(t_k) < 0$$

As a result

$$g^\star = B(1 - \mathcal{U}^K(t_k) - \mathcal{L}^K(t_k)) \qquad \text{with} \;\; g^\star_{t_k} > 0, \; \forall (\alpha, \varepsilon) \in [0, 1]$$